What draws most of us to pursue mathematics are the clarity and purity of the subject, as well as its powers which permeate all of science and thus all of our modern technological world. Somewhat paradoxically, the process of becoming a research mathematician is not as clear and no definitive rules exist which will ensure success. Nevertheless, there are some tried and true ways of guaranteeing a solid foundation for a doctoral education in mathematics. This document lays out some of the minimal expectations of both Ph.D. advisors and their advisees. It contains guidelines and pointers to help the student chart a successful course through the doctoral program in mathematics at Yale, and to help an advisor fulfill their role most efficiently.

1. Advisor’s responsibilities

It is a good idea for a Ph.D. advisor to meet with their doctoral students regularly and on an individual basis. Initially this could mean weekly or biweekly, and as the student enters a more independent research phase, every two to three weeks. One of the most acute dangers facing students in graduate school is isolation. The contact with the advisor is only one way to avoid isolation, or the feeling thereof. Contact with other students working with the same advisor, or in similar fields, can be most beneficial. While these interactions are often established without any faculty involvement, research seminars or group meetings common in the experimental sciences, are an ideal mechanism to facilitate contacts between graduate students and postdocs.
Learning takes place in many ways, and any catalyst which helps the student in the often mysterious process of learning is essential. One of the best ways to learn is to teach. Thus, having students give lectures critiqued by faculty as well as graduate students and postdocs can be a transformative experience.

It is important for an advisor to have clear expectations and to communicate them to the advisee. In many cases, students need guidance about how to think or how to approach a question, rather than solving a particular technical problem. One of the greatest gifts a research mathematician can give to a budding colleague is to show them how they think. At the board and in real time. The student will seek to emulate the advisor, which underscores the importance of excellent graduate courses. In them the graduate students not only learn mathematics, but perhaps more importantly, how to write and present technical mathematics.

As in music, there are form and technique in mathematics. Without technique we are powerless and it is the advisor’s obligation to make sure the student acquires it. This typically means assigning many problems and exercises, and having the student write their work up carefully, and present select problems at the board. This process is especially important early on, but does not lose its relevance throughout a researcher’s career.

A few pointers:

- Think about the timeline: is your student making good progress? Keep the DGS informed if there are any issues.
- Communicate with your student and indicate your expectations.
- Help your student learn how to write. This is hard work: read your student’s writing, and give detailed critical feedback. Find a small writing project early on, long before the thesis. One good way is to ask them to do mini-surveys on problems that they are working on. Also, Graduate Writing Lab has a lot of helpful resources, you may want to direct your students there.
- Make your own plans (travel, leaves) known to your student so that they can plan accordingly.
- Find ways to connect your student to colleagues in the field. Send them to conferences, or to visit your friends and help them find funding for this. Help them find writers of recommendation letters. See the discussion of Years 3-5 below.
- In order to help your student finish in five years, help them plan ahead and monitor their progress starting as early as possible. Note that financial support after year five is not guaranteed by the department, and may be partially your responsibility, see Section 4 for additional details.

2. DGS’ responsibilities

The Director of Graduate Studies (DGS) oversees the graduate program at the Math department. The responsibilities of the DGS related to advising include the following:

- The DGS approves students’ schedules and makes sure that students satisfy the formal requirements of the program.
- The DGS finds faculty members to compose qualifying exams.
- The DGS holds office hours and collaborates with the Graduate Student Advisory Committee (GSAC) to learn about students’ issues and concerns. The GSAC is
a committee that consists of Math department graduate students, and is formed and run by students.

- The DGS facilitates selection of temporary (1st and, in some cases, 2nd year) advisors for students who do not yet have a thesis advisor. This is best done by communicating with students, learning about their interests, and collecting their preferences. The role of a 1st year advisor is to help a student to navigate their graduate studies and provide advice on courses to take, literature to read, faculty to talk to etc. If a preferred 1st year advisor is not available, the DGS may act as such.
- In case there are issues between a student and their advisor, the DGS acts as a mediator. Also the DGS serves as a point of contact when students face personal or medical issues affecting their course of study.
- In case when student’s advisor leaves Yale and the student wants to continue to work with that advisor, the DGS acts as a formal advisor and communicates with the actual advisor.
- Each year the DGS meets with advisors to assess students’ progress.
- It is a good practice for the DGS to have individual meetings with each student to learn about their research interests, progress towards their thesis project, any issues they may have, etc. A good periodicity may be once a semester for students who do not have a thesis adviser yet and once a year otherwise.

3. Student’s responsibilities

Take initiative to read and explore. Ask your advisor for guidance but also explore on your own. Be sure to train your technical abilities by working through problems and exercises. A great way to test your understanding of a theorem or paper is to try to reconstruct the essential arguments a few weeks after you have read it.

You are learning not just to be a student but to be an independent researcher. Meet with your advisor regularly. Initially this could mean weekly or biweekly, and as you enter a more independent research phase, every two to three weeks. You should agree with your advisor on how frequently you meet with them.

Prepare for meetings with your advisor! What this means depends on both of you. Make sure to discuss your respective expectations.

Communicate with your advisor. Do not be afraid to report roadblocks and challenges you encounter (e.g., you are stuck with the proof of a lemma, or an approach you are pursuing does not seem to work, etc.). Those are often the times when you learn the most. Nobody succeeds in proving an interesting theorem at the first attempt, but you need to learn what to avoid, and what methods are more promising than others in certain situations. After a while, you should have the feeling of converging on something, rather than being adrift in a sea of knowledge.

While rare, issues between a student and an advisor can occur. In this case, the DGS acts as a mediator between the student and the advisor unless the DGS is the advisor, in which case the department chair acts as a mediator. If the student feels that they are not going to be successful working with their advisor or do not want to work with the advisor for other reasons, they can consider changing the advisor. This is possible,
although clearly undesirable in later stages of the Ph.D. studies. If a student wants to change an advisor, it is a student’s responsibility to find a new advisor.

Personal or medical issues that affect your study can arise as well. If this is the case, you are encouraged to discuss them with the DGS and/or your advisor. Remember, we are here to help.

3.1. **Teaching.** Part of your training as a mathematician involves learning how to teach. The department has a well-structured program for this, including training in your first and second years and classroom experience after that. Take this seriously! It is worth doing well, important to your career, and can also be a source of enjoyment and satisfaction. Details on teaching duties can be found [here](#).

4. **Timeline for Ph.D. studies**

As a general matter, a student’s time in the program is expected to last five years. Funding is guaranteed for the first five years contingent on satisfactory academic progress and performing the required teaching duties. If students need additional time the department will investigate the possibility of continued financial support, depending on the individual case and availability of funds. Keep in mind that the financial support for the sixth year and beyond is not guaranteed: its level depends on how many slots we have for TAships and the funding of individual faculty. In view of this it is important to remain aware of the passage of time and of the need to take care of important milestones.

Students should be aware of the requirements of the program that are summarized [here](#).

4.1. **Year 1.** You will expend significant efforts studying towards your qualifying exams, and probably take some or all of them by the spring. At this point, there are three exams: Algebra, Analysis, Topology. Looking at the past exams and talking to fellow students about their experiences should help you prepare. Check the department website for more information including syllabi for the exams.

- Get to know your respective first year advisors and talk to them about available research areas, so that you may start to focus on your interests.
- Attend seminars. Even in the first year start to spend time in talks. Don’t be discouraged if you do not understand much, remember that even faculty members often don’t understand everything.
- Get to know your fellow students, and organize some study groups or seminars. In the past, we have found studying in groups to be a very effective way of learning complex material. Your fellow students are also the best source for information about everyday life (where to rent, where to shop, how to buy cheap tickets etc). Reach out to them.

4.2. **Year 2.** Complete any remaining qualifying exams and the two honors grade requirement by the end of the second year. Continue broadening your outlook, but also focus on a research area. Try to talk to as many faculty members as possible, to see what they are actively doing, which may be different from what you saw online. In particular, this is a good way to identify possible advisors. To facilitate communication with the faculty, in recent years, the graduate students organized a series of mini-colloquia, where faculty members speak about their research. Do attend those – usually, the food is served
– and don’t be shy to ask questions. The goal of these talks is to serve you! Audiences are restricted to graduate students.

By the end of the year, you should be selecting an advisor and looking for a thesis problem, or at least a definite direction. Do not postpone asking faculty members whether they agree to advise you until the last moment, you may discover that they are unable to do so for a number of reasons: they have many students already, plan to retire or leave soon, etc.

4.3. **Years 3,4.** You are working on your thesis problem. Try to consult not only with your advisor, but also with fellow students and postdocs, and researchers outside of Yale. Making connections outside the department can be very beneficial for you as you look for your first position after graduating.

Note that you are required to submit the dissertation prospectus during the third year (see here) and fill the annual dissertation progress reports until the dissertation itself is submitted. The prospectus should give a brief introduction into your general area of research, outline the existing developments and describe what you plan to do for your thesis. If you already obtained some results on your project, the prospectus should mention that as well. Also you need to satisfy the eight course requirement by the end of the third year.

Once you have results on your thesis work, it is essential to present them more broadly especially if you plan to stay in academia. Your advisor may be able to help with an invitation for you to speak at a seminar or a conference. One possible venue for graduate students to give talks are the AMS meetings. They often invite proposals for talks. Check the AMS webpage for more details.

Aim to have enough results for a thesis by the end of your fourth year. That’s when you start asking for letters of recommendation.

When you have results, you should write them up. Your advisor should be able to give you advice on how to write research papers. Upload the text to arXiv so that mathematicians can learn about your work, and then submit your paper to a journal. Having a published paper may help you with your academic job search. Talk to your advisor regarding journals to submit your paper to.

It is also a good idea to create a however basic professional webpage, where you can post your CV and links to your papers or other texts. This increases your visibility and may provide some help in finding a job in later years.

4.4. **Year 5.** In the fall you are applying for jobs. The deadlines usually start in November. You should have recommendation letters written and submitted in time. Keep in mind that many letter writers are very busy during this season. So it makes sense to send requests for letters at least one month in advance.

You will finish writing your thesis and submit it in early spring. See here for the procedure. You should also find a second reader for your thesis – preferably early in the year so they have time to read it in detail.

For job applications, it is a good idea to contact people you know outside of Yale (see Year 3-4) and tell them that you are applying. You can also reach out to people working in the general area that you are interested in, even if you have not met them. Introduce yourself, and send them a research description (what you have done and plan to do in
the next few years) and also a CV. Do not be shy. Most senior researchers really want to help junior people to a good start. Usually you will know about the result of your job application in January and February, but in some cases (concerning waitlist, for instance), it can be as late as April.

The Ph.D. defense typically takes place in the spring semester of the final year. You should start to organize and schedule your defense early in the spring semester, ideally, by the end of February. The defense is an opportunity for you to consolidate your work and present it, in an official way, to your peers and to the faculty. It is also a celebratory occasion, providing an opportunity for your colleagues, friends and family to witness and share in your accomplishment. The department has allocated some funds so that you can provide refreshments for your guests. You have worked hard enough, so do not forget the fun!

4.5. **Year 6 and beyond.** In some cases, the program can take more than five years. In that case, you need to adjust the process in your fifth year accordingly. The earlier your advisor and the DGS know that you plan to stay for the sixth year, the better. A good time to discuss this with your advisor is the beginning of the fifth year.

5. **Some Career Advice: Beyond Academia.**

You can do many things with a PhD in mathematics. Being a professional mathematician is one of them, but not the only one. Many of our students are now working in industry, in very good companies or research labs. To prepare for such a career, it is usually a good idea to try an internship in industry in the summers, and have proper programming skills. Taking additional courses in necessary areas in or outside of Math should help as well. Knowing how to communicate with people outside your specialized research area is also important. Yale’s Office of Career Resources provides a lot of useful resources for finding jobs in industry including how to write resumes, how to prepare for interviews, etc.

Once you have an open mind, the possibilities are endless.

6. **Odds and ends**

*Travel funding.* The department has a fund to support your travel for research purposes. You can use it in any year, but perhaps it is a good idea to use it in later years (starting, say, year 3) when you have enough work done and experience. The total amount is currently 2000 USD per student, but you should check with the DGS about regulations before making traveling arrangements.

*Questions and issues.* There are a number of people in the department you can go to if you have a problem or a question. Your advisor, of course and your first-year advisor can help with mathematical questions, advice about research directions and so on. For general questions about the program, or help navigating the graduate school bureaucracy, you can talk to the DGS and to the department Registrar. You can go to the Chair with questions that are perhaps more sensitive or that seem to require a broader outlook. And of course all faculty members are happy to talk to you and find ways to help. You can also consult with the Graduate Student Advisory Committee (GSAC), which is a group of graduate students acting as liaison between the students and the faculty.
One good and important place to bring questions/concerns to is the Town Hall meeting. The Chair and DGS will hold such meetings with the graduate students at least once every year. In these meetings, you can discuss ways of making the department a better place for students. You can raise any issue – research, courses, social life, building problems – whatever is important to you.

And finally: remember that there is more to being a complete human being than mathematics. Make time to do things that you love, and to build and nurture your relationships with friends and family. All of us recognize the importance of this and leave room for it in our lives.

7. Some helpful links

• The graduate school page on advising and mentoring has much more information.
• AMS Graduate student blog.
• How to get a PhD in a timely fashion, Sara Billey.
• Career advice by Terry Tao.
• Career information from the AMS.